PROOF OF NASH PROBLEM FOR SURFACES.

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Nash formulated this problem in an attempt to understand resolution of singularities of a variety X in relation with the space of arcs in X centred at the singular locus. The space of arcs is an infinite dimensional algebraic variety given by the inverse limit of the spaces of n-jets, which are finite dimensional algebraic varieties. Consider a resolution of singularities of X and take the decomposition of the exceptional divisor $E = \bigcup_i E_i$. Given any arc $\gamma : (\mathbb{C}, 0) \to (X, SingX)$ one can consider the lifting $\tilde{\gamma} : (\mathbb{C}, 0) \to (\tilde{X}, E)$. Nash considered the set of arcs whose lifting $\tilde{\gamma}$ meets a fix divisor E_i, that is $\tilde{\gamma}(0) \in E_i$ and proved that these are irreducible sets of the space of arcs. Nash's question is whether for the essential divisors E_i they are in fact irreducible components of the space of arcs or not (an *essential divisor* appears by definition in any resolution of X up to birational mapping). He conjectured that the answer was yes for the case of surfaces (for which there exists a minimal resolution that has only essential divisors) and suggested the study in higher dimensions. In 2003, Ishii and Kollar gave an example of a variety of dimension 4 for which some of these sets are not. Hence the case of dimension 2 and 3 remained opened.

We solved the conjecture for the surface case in a joint work with J. Fernández de Bobadilla. In the talk I will give an introduction to the problem and sketch the proof for the surface case. After works of M. Lejenune Jalabert, A. Reguera and J. Fernández de Bobadilla the problem deals with holomorphic 1-parameter families of convergent arcs. The key of our approach is to work with representatives of appropriate arc families and find a topological obstruction to their existence. The obstruction is expressed as a bound for the euler characteristique of the normalization of the representative of the generic member of the family, which we know is a disc.

After our proof, counterexamples in dimension 3 were found by T. de Fernex and a generalization for higher dimensions (including another proof for the surface case) was given by T. de Fernex and R. Docampo. They prove that exceptional divisors that appear in any minimal model with terminal singularities give irreducible components of the space of arcs. The conjecture about what other divisors give irreducible components has still to be stated, for example, for terminal singularities is completely open.