Hasse-Schmidt derivations : an interesting notion to play with in non-zero characteristics

If A is a commmutative algebra over a commutative ring k, a Hasse-Schmidt (HS) derivation of length m is a k-algebra map $D : A \to A[[t]]/\langle t \rangle^{m+1}$ such that $D(a) \equiv a \mod t$. We may understand HS-derivations as "arc fields". Given a HS-derivation $D(-) = \sum_i D_i(-)t^i$, its 1-component D_1 is always a k-derivation of A. A basic question is whether any k-derivation of A cam be extended to a HS-derivation of length m, for some, or for all m, or for $m = \infty$. This is always true if k contains the field of rationals, but it is far from being true otherwise. The second basic question is to understand the modules of "m-integrable derivations" of our k-algebra A, denoted by $\mathrm{IDer}_k(A; m)$, and to relate them to other invariants of our "singularity" A over k.

In this talk we will recall some of the basic known results about HS-derivations : its relationship with ring of differential operators, their use in describing differential smoothness in a general setting and, in such a case, how a structure of D-module can be recovered by the action of HS-derivations.